OPEN ACCESS

European Journal of Engineering Science and Technology
ISSN 2538-9181

Simulations and Results for the Heat Transfer Problem

Shkelqim Hajrulla*, Arban Uka, Taylan Demir

Computer Engineering Department, Epoka University, Tirana, Albania

ARTICLE INFO

Keywords:
Transforms,
decomposition method,
image rendering,
Poison equation,
Distribution

ABSTRACT

In many studies of the heat transfer problems, many researchers have studied the heat transfer problems by solving partial differential equations with the use of approximations or finding the solutions with experimental data. In this paper, we analyze the heat transfer problem using a heat source in a closed environment and how it transfers in the neighboring sections. We refer to mathematical concept to make possible the simplification of the complexity that associates such thermodynamic problems. We will discuss such a problem as a discrete one, easily computable, rather than treating it as a continuous one. We have reduced this problem into solving a simple system of linear equations and differential equations. In many cases differential equations are hard and difficult to solve. So, we use numerical methods to approximate the differential equations to algebraic equations and solve them. We will compare different algorithms used and show which one of them performs better under our test conditions. The program will simulate the heat transfer of a single heat source in a closed environment. The results of the simulations will be presented in graphs and demonstrated in visual settings. In the end, we will provide our conclusions on the performance of the numerical methods. We achieve the purpose of this study, which is to analyze the heat generation analysis and heat transport in three-dimensional space as to the neighboring sides of a closed environment.

1. Introduction

In the theory of heat transfer it is very important to understand that heat transfer involves many differential equations. Considering computational runtime, we are able to apply the numerical methods and the error due to approximation we obtain with numerical method is acceptable. We deal with the study of algorithms that have been used to approximate the solutions of complex problems. Most of the problems do not have an analytical solution and thus the optimal use of numerical techniques becomes essential. One the most fundamental useful mathematical constructs are the matrices. Almost all the operators can be represented by matrices including derivatives of several orders, Fourier transforms etc.

* Corresponding Author E-Mail Address: arbanuka@gmail.com

Cite this article as:
Hajrulla, S., Uka, A. & Demir, T. (2023). Simulations and Results for the Heat Transfer Problem. *European Journal of Engineering Science and Technology, 6*(1): 1-9. https://doi.org/10.33422/ejest.v6i1.986

© The Author(s). 2023 **Open Access**. This article is distributed under the terms of the Creative Commons Attribution 4.0 International License, which permits unrestricted use, distribution, and redistribution in any medium, provided that the original author(s) and source are credited.

The use of matrices is in principle computationally expensive, but the use of large computational units overcomes this barrier. In the fundamental techniques that computers operate, only the addition is possible and the rest such as subtraction, multiplication, and division are defined in elegant digital design methods.

Other operations like finding the square root of a number, solving a system of linear equations, solving a differential equation are made possible only through the use of numerical algorithms (Harbrecht & Multerer, 2022) that can be studied in mathematics, studing elements of finite method (Yang, 2018), an efficent solving method for nonlinear convection diffusion equation (Cui et al., 2018) and numerical analysis of one dimensional heat transfer (Mahardika & Haryani, 2020).

In this paper, we have discussed about how to determine the heat transfer in a closed environment from a heat source located at the center of the region. Here we consider three assumptions: i) analyses is conducted in a 2-dimensional space, ii) steady-state is considered and, iii) no heat transfer with the surrounding is considered (closed system).

The different frequencies, amplitudes and phase angles of sinusoidal heating are used as for understanding the heat behavour problems and standard for Interconnection of distributed energy resources is studied (Feynman, 2018), (Manna et al., 2019, pp. 3822-3856). An overview for thermal characteristics (ISO 13789) and used thermal coefficients of buildings-trasmission heat transfer problem is studied (ISO 13786).

We first start with a basic explanation of Laplace- operator. We can obtain the results using the unimodular matrix through python (Hajrulla et al., 2021, pp. 134-141), (Hajrulla et al., 2022, pp. 45-55). For the same, we mention that the Laplace operator is a differential operator given by the divergence of the gradient of a scalar function on Euclidean space. In Cartesian coordinate system, it is denoted by Δ. We can say that this operator is used for describing many physical phenomena, which involve differential equations.

An efficent solving method for nonlinear convection equation is presented and algorithm for improving power balance is mentioned (Kunzler, & Lopes, ICEON 2019, pp. 6066-6071), as well as power balance technique for cascaded H-bridge multilevel cells in a hybrid power amplifier in a simplified way (Kunzler, & Lopes, ICIT 2018, pp. 800–805).

2. Materials and methods

2.1. Laplace-operation and transforming the heat equation

In this paper we will be based on the method used in Alshmary (2020) which even though uses the Poisson’s equation, which is normally used for analyzing the electrostatic potential in a room with electrons, it is possible to approximate the heat distribution of a heat source in a closed room on a 2-dimensional space by using same form of Poisson’s equation. The equation that we will be based on will be this:

$$\Delta u(x) \coloneqq \frac{\partial^2 u}{\partial x_1^2}(x_1, x_2) + \frac{\partial^2 u}{\partial x_2^2}(x_1, x_2) \quad (1)$$

Our goal consist in separation of the two parts of this equation and study then in a discrete form. Based on Harbrecht and Multerer (2022) we say that:

$$\frac{\partial^2 u}{\partial x_2^2}(x_1, x_2) \approx \frac{u(x_1,x_2+h)-2u(x)+u(x_1,x_2-h)}{h^2} \quad (2)$$

For analogy, we can say that:

$$\frac{\partial^2 u}{\partial x_1^2}(x_1, x_2) \approx \frac{u(x_1+h, x_2) - 2u(x) + u(x_1-h, x_2)}{h^2} \tag{3}$$

We can select a number $n \in N$ and select a unit square of Ω followed by equally distanced spaces of $h = 1/n$. The grid points are than given by:

$$x_{i,j} = h[i,j]^T \quad for\ all \quad i,j \in \{0,1 \ldots n\} \tag{4}$$

Based on equations Eq. 2.2 and Eq. 2.3 our approximation on Poisson's equation for every value of every cell in our space will be given by the expression:

$$\frac{1}{h^2}[4u_{i,j} - u_{i-1,j} - u_{i+1,j} - u_{i,j-1} + u_{i,j+1} = f_{i,j} \tag{5}$$

As a result, we can transform our equation in a system of linear equations expressed as $Au = f$ where matrix $A \in \mathbb{R}^{N \times N}$ where $N = (n-1)^2$.

The system of equations is shown below:

$$Au = \begin{pmatrix} 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & -1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 4 & 0 & 0 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 4 & -1 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 4 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 4 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & -1 & 4 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 4 \end{pmatrix} \begin{pmatrix} u_{1,1} \\ u_{1,2} \\ u_{1,3} \\ u_{2,1} \\ u_{2,2} \\ u_{2,3} \\ u_{3,1} \\ u_{3,2} \\ u_{3,3} \end{pmatrix} = \begin{pmatrix} f_{1,1} \\ f_{1,2} \\ f_{1,3} \\ f_{2,1} \\ f_{2,2} \\ f_{2,3} \\ f_{3,1} \\ f_{3,2} \\ f_{3,3} \end{pmatrix} \tag{6}$$

In addition, the corresponding point of this equation on a graph would be:

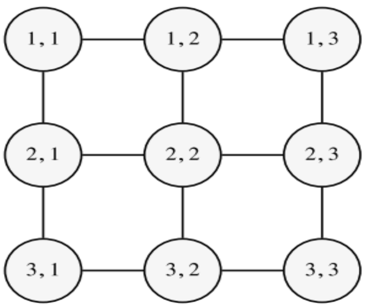

Figure 1. Representing the corresponding position of every U number

So, our goal now has become solving this system of linear equations, so we can find the value of temperature that every cell in our grid will have in a specific period. As we can observe, we have transformed this complex problem into a problem that involves several linear equations. This is much efficient for the computer to calculate but still special algorithms, which we will be discussing in section 3, should be used.

2.2. The initial state of the system

To solve our problem, we will first need to give some initial heat values to our closed system, which is an initial heat source, from which heat will be released and send to the other parts of the system (2d room).

If we were to be based on the analyses discussed (Yang, 2018) we could represent the heat source as a function of sin(x) in a 3-dimensional space, such as:

$$f(x, y) = \sin\left(\frac{\pi x}{n}\right) + \sin\left(\frac{\pi y}{n}\right) + b \quad (7)$$

Where b is an arbitrary number that we have chosen to represent heat sources with different capabilities. The larger value for b indicates the larger energy released by the heat sourse. Nevertheless, as stated this is not a perfect representation of how a heat of a heat source would be distributed. Therefore, we go with the second alterative given by assuming that the heat is uniformly distributed along the heat source (Yang, 2018; Feynman, 2018).

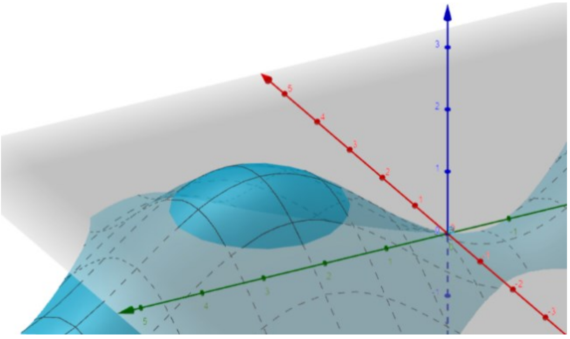

Figure 2. Showing how we can model the heat distribution of a heat source

3. Application

To solve this problem, we would need to solve the system of linear equations that we have formed in section 1.2. To do so we will be using 2 different mathematical methods that can be used to solve such systems. The first one would be Gauss-Seidel method and LU-Decomposition method.

3.1. Generating the [A] matrix and using numerical method

Before starting to solve our system of linear equations, we would need to generate the pattern like matrix A. generating the matrix A, we now need to perform the calculations for solving the system of linear equations.

To do so we have used the Gauss-Seidel method. This is an iterative method, consisting in solving a system of linear equations by first getting an initial value and then performing the algorithm recursively until an initial absolute error value is reached. We also need to keep in consideration the absolute error.

We would be able to solve this system of linear equations by doing such calculations:

$$x_i = \frac{b_1 - (a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in-1}x_{n-1})}{a_{in}} \quad (8)$$

The method that we have implemented in the code has been modified so that it allows us to perform more iterations to get a better result. The code written in C++ is given in appendix.

The second method that we will be discussing is LU Decomposition. It mainly consists of decomposing the A matrix and then carrying out the calculations such that it is more efficient for us to perform multiplication with L and U matrix, than it was with A matrix.

In our C++ code we have done a mixing of Gauss-Seidel method with LU Decomposition method. More specifically, we have performed the LU decomposition with a specific algorithm and after the decomposition of matrix A into matrixes L and U we have used the Gauss-Seidel method to solve the two new systems of linear equations.

3.2. Performance comparison

After giving the code of both methods we need to analyze which one of these methods performs better in our test conditions. The measurements have been carried out in an HP laptop, Intel i3 gen 8^{th} processor, with 8 GB DRAM. The result that we got are shown by a tabular representation.

We have considered four test cases. After performing the measurements, we ended up with the following results:

Table 1.
For the Gauss-Seidel method, results as follows

Case	**1**	**2**	**3**	**4**
Time	1999000 mls	3002000 mls	27000000 mls	9990000 mls
Number of iterations	13	5	15	16

Table 2.
For the LU Decomposition method, results as follows

Case	**1**	**2**	**3**	**4**
Time	1004000 mls	792300032 mls	172023186 mls	35000000 mls
Number of iterations	8	2003	8	8

What we observe by the results is that the LU decomposition method has a better performance that is almost constant when we are talking about the number of times called to solve the equation.

Nevertheless, it has the risk of diverging as we see in case 2 where we have reached the number of calls limit, set by us, and the time which shows overflow of the information.

On the other hand, we see that Gauss-Seidel method has a better time in almost every case compared to LU decomposition method. An important fact to be mentioned is that we have taken as a constant number of iterations the time it takes the LU decomposition algorithm to perform the decomposition process.

4. Simulation

In this part of our paperwork, we have given a simulation to show the result of the method that we have discussed in section 2. To perform the simulation, we have used C++ along with a specific library of it, namely SDL2. We have performed two simulations with different heat sources and different resolution size.

In the first one we have given example, which is relevant with those explained in section 2. The resolution of the image is 3 by 3. A heat source of 100 grade Celsius is placed in the center of the room. As we can see the heat has been released in a symmetric way in every part of our room.

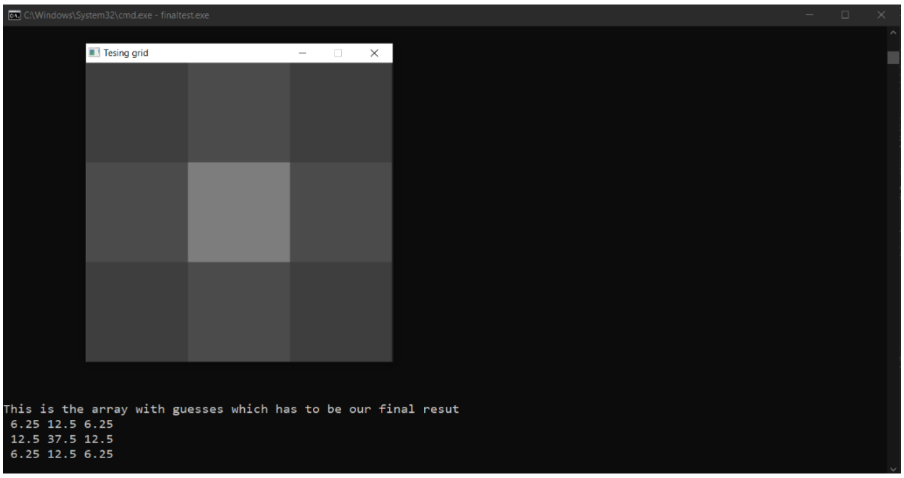

Figure 3. The first simulation with a 3 by 3 matrix

In the second simulation, we have increased the number of squares to 7 that will represent the room. Along with that, there are three points that represent e heat source of 100 grade Celsius in the middle of the room. As we can see the middle of the room is much warmer that the rest of the room since the heat source is placed in a horizontal direction.

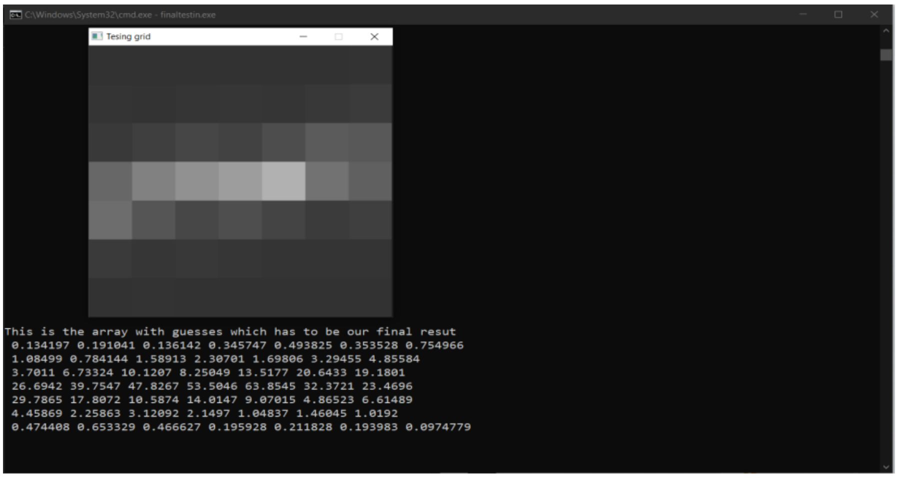

Figure 4. The second simulation with a 7 by 7 matrix

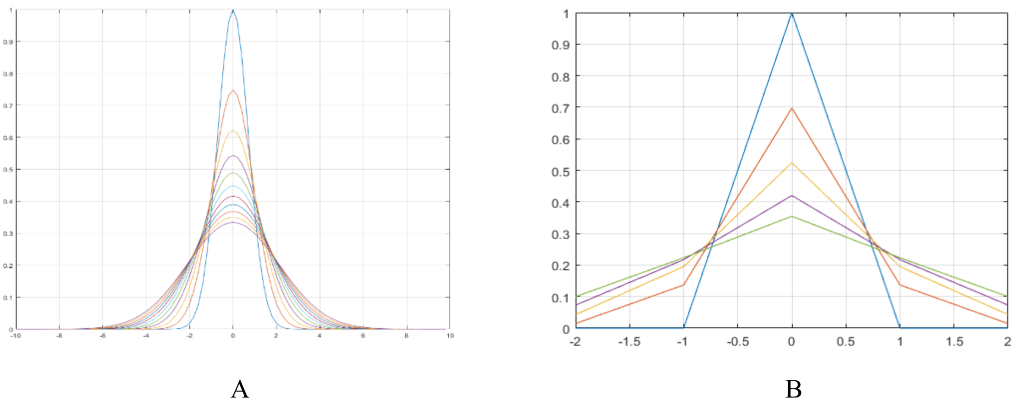

Figure 5. Heat distribution for the first simulation (A left) and Heat distribution for the second simulation (B right)

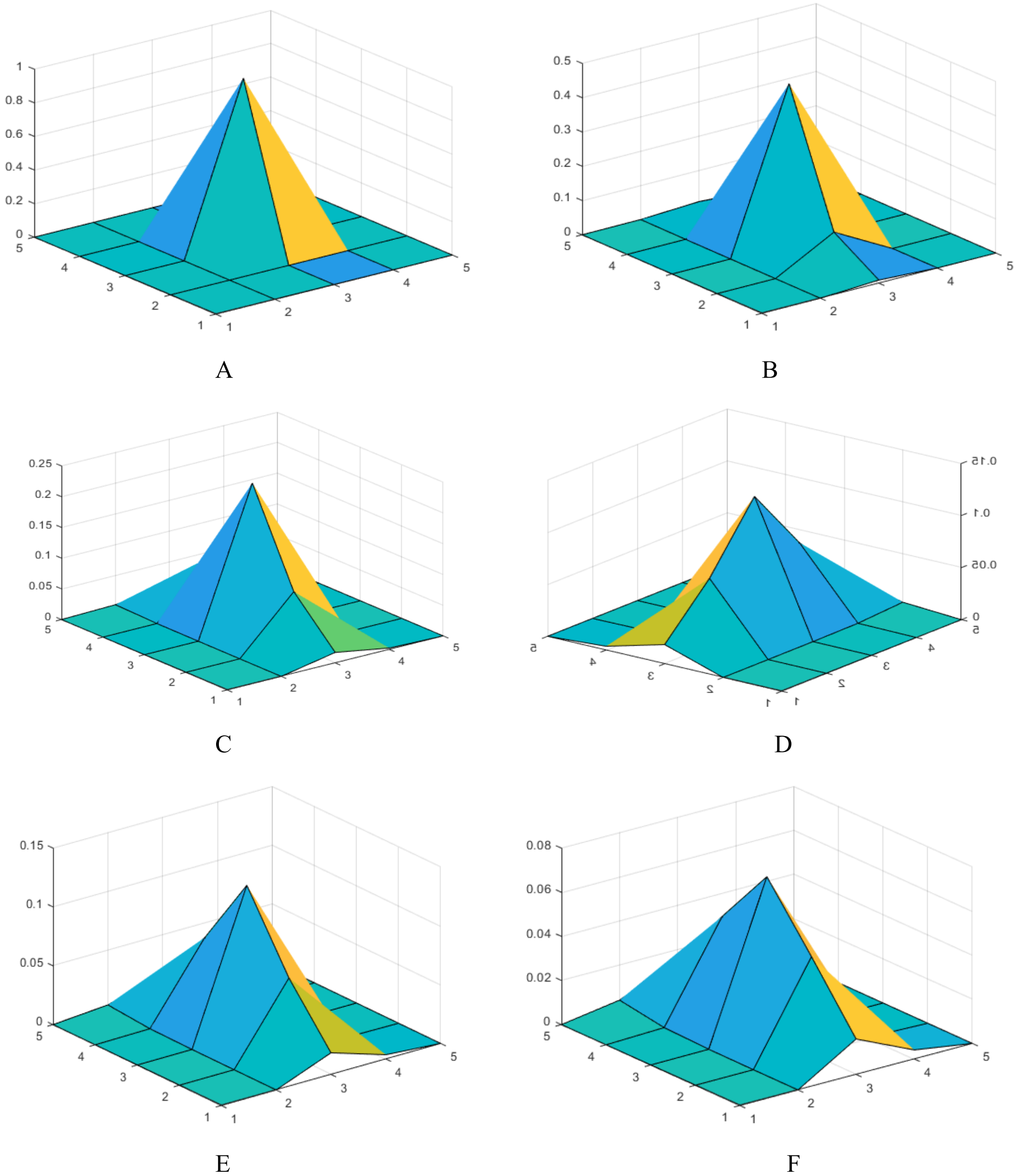

Figure 6. A, B, C, D, E, F (from the left to the right): Comparison of heat distribution in different conditions

5. Conclusions

The heat transfer problem from our point of view using multi-frequency heating is estimated under different methods. The effect of different frequencies of sinusoidal heating, along with the uniform heating, is discussed using the simulations, numerical methods and approximations. This is performed while using thermal boundary layers and heatlines. The simulation results show the importance of using their code to heat transfer problems in a closed environment and found to have good results in explaining the heat transfer behavior on separated particles of area.

The numerical methods are used to solve the heat transfer problem, considering the methods and comoutional operations, approximation and simulation model. The mean-temperature constraint is applied for enhancement analysis. The approaches are quite effective on image restoration using matrices of different dimensions. Similar studies were carried out as well, but we used the test evaluation based on numerical methods.

We can say that it is possible with the help of numerical results to simplify problems in a way that it is efficient for us to compute the result. Computers have limitations concerning the number of operations they can perform. Nevertheless, in most of the cases we end up with a problem that requires many calculations.

Therefore, with the aid of computers, we can avoid these errors and the long amount of time needed to perform the calculations. Numerical methods help us to perform complex calculations of complex heat-transfer problems. The algorithm has been selected and perhaps finds one more relevant for the heat transfer of an object. The used algorithm indicates such results as an optimisation technique by computing a single graph.

In the next future, the different frequencies, amplitudes and phase angles of sinusoidal heating will be investigated to understand their major impacts on the heat transfer characteristics. The present research idea can usefully be extended to other multi-physical areas (nanofluids, magneto-hydrodynamics, etc.). Also, the next study will focus on energy minimization methods for finding a minimum on the initial potential energy surphace.

Theoretical analysis shows that each of the schemes is stable, its solution exists uniquely and has temporal accuracy. The corresponding iteration number has the same order of accuracy and quadratic convergent speed. Numerical tests verify the conclusions and demonstrate the high accuracy and efficiency of the algorithms.

The numerical method provides theoretical and technical support to accelerate resolving heat diffusion, heat transport problems, approximation of solutions and expected results. We develop a numerical model of analyzing the heat transfer problem using a heat source in a closed environment and how it transfers in the neighboring sections. The temperature distribution inside space was compared and the results was shown in table data and in the graphically form.

Acknowledgement

We are grateful to International Organization for Standardization (ISO) publications. This work would not have been possible without the support of Doctoral School of Epoka University. We are grateful to all reviewers of our article.

Future work

Based on (Manna, Biswas, & Mahapastra, 2019), we will study the efficent solving method for nonlinear convection diffusion equation and converting the proper equation as for the heat transfer problems. Of course, this would be a challenge for us, difficult situation because of the combination of conservative diffusion and convection operator. Furthermore, the algorithm would be extended from first-order temporal accuracy to second-order temporal accuracy. Of course, using numerical methods, appropriate algorithms and simulations, we can achieve our goal of future studies related to heat transfer problems.

References

Alshmary, R. M. H. (2020). *"Application of Partial Differential Equations"*, Journal of Physics: Conference Series, J. Phys.: Conf. Ser. 1591 012105. https://doi.org/10.1088/1742-6596/1591/1/012105

Cui, X., Yuan, G. and Shen, Z. (2018). "*An efficient solving method for nonlinear convection diffusion equation"*, International Journal of Numerical Methods for Heat & Fluid Flow, Vol. 28 No. 1, pp. 173-187. https://doi.org/10.1108/HFF-10-2016-0403

Feynman. (2018). *"IEEE Standard for Interconnection and Interoperability of Distributed Energy Resources with Associated Electric Power Systems Interfaces",* IEEE-SASB Coordinating Committees: Piscataway, NJ, USA.

Hajrulla, Sh., Arban, U., Bezati, L., Demir, T, & Desantila, H. (2022). *"Unimodular matrix on shallow water wave theory".* pp. 134-141 ISBN: 978-605-69181-5-5 Online International Symposium on Applied Mathematics ISAME 2022, Istanbul.

Hajrulla, Sh., Bezati, L., Demir, T., & Desantila, H. (2021). *"Through Unimodular Matrix on SLE using LaTex. "* pp. 45-55 December 2021, Istanbul. International Conference on Mathematics.

Harbrecht, H., & Multerer, M. (2022). *Algorithmische Mathematik Graphen, Numberik und Probabilistik*, Springer Spectrum (e-Book) (pg. 167-168). https://doi.org/10.1007/978-3-642-41952-2_9

International Organization for Standardization (ISO). (2017). *"Thermal Performance of Buildings–Transmission and Ventilation Heat Transfer Coefficients—Calculation Method"*; ISO 13789; International Organization for Standardization (ISO): Geneva, Switzerland. [Google Scholar].

International Organization for Standardization (ISO). (2017). *"Thermal Performance of Building Components—Dynamic Thermal Characteristics—Calculation Methods"*; ISO 13786; International Organization for Standardization (ISO): Geneva, Switzerland. [Google Scholar].

Kunzler, L.M. & Lopes, L.A.C. (2018). *"Power balance technique for cascaded H-bridge multilevel cells in a hybrid power amplifier with wide output voltage range",* In Proceedings of the 2018 IEEE International Conference on Industrial Technology (ICIT), Lyon, France, 20–22 February 2018; pp. 800–805. https://doi.org/10.1109/ICIT.2018.8352280

Kunzler, L.M., & Lopes, L.A.C. (2019).*"Algorithm for Improving Power Balance for Cascaded H-Bridge Multilevel under Staircase Modulation for Linear Loads"*, In Proceedings of the IECON 2019-45th Annual Conference of the IEEE Industrial Electronics Society, Lisbon, Portugal, 14–17 October 2019; Volume 1, pp. 6066–6071. https://doi.org/10.1109/IECON.2019.8927489

Mahardika, D. P. and Haryani, F. F. (2020). *"Numerical analysis of one dimensional heat transfer"*, Journal of Physics: Conference Series, J. Phys.: Conf. Ser. 1591 012105 J. Phys.: Conf. Ser. 1511 012049. https://doi.org/10.1088/1742-6596/1511/1/012049

Manna, N.K., Biswas and Mahapastra, P.S. (2019). *"Convective heat transfer enhancement: effect of multi-frequency heating"*, International Journal of Numerical Methods for Heat & Fluid Flow, Vol. 29 No. 10, pp. 3822-3856. https://doi.org/10.1108/HFF-07-2018-0414

Yang, K. H. (2018). *"Basic Finite Element Method as Applied to Injury Biomechanics".* https://www.sciencedirect.com/topics/engineering/gauss-seidel-method