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Real Options Approach for Technology Renewal in an Electronic Manufacturing Service (EMS) Company

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ABSTRACT

Electronic Manufacturing Services (EMS.) companies are in a highly competitive environment where investment in technology plays a significant role in the company's performance. The equipment efficiency is directly converted to revenue. The equipment efficiency decays over time; keeping the equipment represents a loss in productivity, and renewing the equipment requires an additional cost. There is a continuous decision process to determine the optimal time to replace the old equipment with a new one. Traditionally, the optimal time to renew is when the machine's revenue matches the cost. This paper studies the renewal decision using a real options approach, adding the uncertainty factor. The variables are modeled as Geometric Brownian Motions. We provide a literature review of renewal real options and describe the models we use. The one-factor model considers the revenue as stochastic; the two-factor model considers the revenue and cost as stochastic; the technological improvement models extend the two-factor model to include a premium in revenue for replacing the equipment. An overview of the electronic manufacturing dynamics is described; we select a product whose manufacturing process depends on machines. We provide a methodology for the model's implementation and how to determine the parameters; the results are compared to the deterministic approach. Finally, we discuss the models' advantages, disadvantages, and limitations.

1. Introduction

The concept of renewal technology has been present since the creation of tools in the stone age. Ancient ones should have decided when to replace old tools and when to adopt a different technology. A simplistic approach is to renew equipment when the old one is broken; however, the problem is more complex. The machines and tools do not suddenly fail. There is gradual decay in efficiency as time passes; this implies that revenue decreases and cost increases;

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revenue and cost fluctuate, and the variation is different in each case. Furthermore, technology progress fast, making available improved products in a short period of time.

Several authors have studied the optimal renewal time for technology. some studies of renewal are dated from the industrial revolution, Faustman determined the optimal time for harvesting and cutting trees in (Linnard & Gane, 1968); the model assumes that the equipment's revenue decreases and the cost increases with the usage at a linear rate without fluctuations. (Terborgh, 1949) considers the cost as dynamic with a time dependency. The discounted cash flows and taxes were added by (Eilon, et al., 1966).

(Tourinho, 1979) was a pioneer using options to model the uncertainty factor in the resource extraction business. (McDonald & Siegel, 1986) continue the work evaluating business opportunities. (Dixit, et al., 1994) created a framework to include the uncertainty in business decisions. In a renewal decision, the revenue and the cost of equipment are stochastics. (Dobbs, 2004) consider one of the factors as stochastic, modeling the cost or the revenue as a Geometric Brownian Motion. (Margrabe, 1978) uses two stochastic variables to propose closed-form solution for an exchange option, (Geske & Shastri, 1985) and (Cortazar, 2000) added more stochastics factors, but the solution relies on numerical approximations. (Caballero & Pindyck, 1996) and (Paxson & Pinto, 2005) use two stochastic factors model proposing a closed-form solution under certain conditions. (Adkins & Paxson, 2011) use a two-factor model in the renewal decision.

Prior models consider the technological replacement with the same technology, i.e., get the exact same machine but brand new. (Caplan, 1940) considers a premature renewal caused by a technological improvement. (Stapleton, et al., 1972). The prior machine is replaced with an improved one. (Adkins & Paxson, 2013) (Adkins & Paxson, 2014) extended the two-factor model to consider technological progress.

Four models are selected for implementation. In the first section, we describe the deterministic model (Linnard & Gane, 1968); the one-factor model (Dobbs, 2004); the two-factor model (Adkins & Paxson, 2011); and the two-factor model with technological progress (Adkins & Paxson, 2014). We examine the models and guide the implementation. In the study case, we present the electronics industry's general dynamics and explain the importance of optimal renewal in the sector. We compare the results across the models and discuss their advantages, disadvantages, and limitations using sensitivity analysis for the critical parameters. Conclusions are drawn at the end of the paper

2. Methods

The Net Present Value (NPV) theory considers the value of future cash flows using a discount rate. The uncertainty factor is included in the discount rate, which is assumed constant. The real options theory extends the NPV approach by considering the cash flows as stochastics and commonly models as Geometric Brownian Motions (GBM). The renewal option is the option to replace equipment or any asset with another; the option is applicable where the asset is perceived to deteriorate with time or continual usage, which affects the revenue and cost of the company. The option is exercised at the right time and at the right value to reduce costs and maximize revenue. Renewal options are used mainly in businesses such as transportation, hotels, airlines, and in any industry where equipment represents the central operation process. We select four representative renewals models to compare them; the nomenclature and guide for implementation is based on notes from (Paxon, 2013).

2.1 Deterministic model

The deterministic model assumes that the revenues (P_1) and the costs (C_1) are constant; therefore, their volatilities are nil ($\sigma_p = 0;\ \sigma_c = 0$). The first order condition gives the optimal revenue ($\hat{P}$) and cost ($\hat{C}$) for the maximum NPV for the machine, i.e., at the optimal renewal time, the present value of old equipment is equal to the net present value of the replacement investment:

$$\hat{P}\left(\frac{1}{r}+\frac{\theta_p}{r}\times\frac{e^{-r\hat{T}}}{r-\theta_p}\right)-\hat{C}\left(\frac{1}{r}+\frac{\theta_C}{r}\times\frac{e^{-r\hat{T}}}{r-\theta_C}\right)=\frac{P_1}{r-\theta_P}-\frac{C_1}{r-\theta_C}-K \qquad (1)$$

, where $\hat{T}$ is the optimal cycle time, which is calculated using the following three formulae:

$$\hat{T}=\frac{1}{\theta_c}\ln\left(\frac{\hat{P}}{C_1}\right)=\frac{1}{\theta_p}\ln\left(\frac{\hat{P}}{P_1}\right)\ ;\ \theta_p\beta+\theta_c\,\eta\ -r=0\ ;\ \left(\frac{P_1}{\hat{P}}\right)^{\beta}\left(\frac{C_1}{\hat{C}}\right)^{\eta}-e^{-r\hat{T}}=0 \qquad (2)$$

, where P_1 and C_1 are the revenue and the cost at the original level of the brand-new equipment; θ_p and θ_c are the revenue decline rate and cost increase rate; K is the investment cost; and r is the interest rate; β and η are auxiliary variables, we find their value solving the equations.

2.2 One-factor renewing model

In the one-factor model, we use (Dobbs, 2004) considering the revenue as follows a geometric Brownian motion. The cost volatility and drift is nil. $\sigma_c = 0, \theta_c = 0$; there is no correlation between the revenue and the cost, $\rho = 0$. The revenue threshold level is given by:

$$\frac{\hat{P}}{\beta_1(r-\theta_p)}\left(\beta_1-1+\left(\frac{P_1}{\hat{P}}\right)^{\beta_1}\right)-\frac{P_1}{r-\theta_p}+K=0 \qquad (3)$$

$$\beta_1=\left(\frac{1}{2}-\frac{\sigma_p}{\sigma_p^2}\right)-\sqrt{\left(\frac{1}{2}-\frac{\sigma_p}{\sigma_p^2}\right)^2+\frac{2r}{\sigma_p^2}} \qquad (4)$$

, where P_1 is revenue at t=0 and $C_1$is cost at t=0; K is the renewal investment; θ_p and θ_c are percentage rate of changes in the revenue and the cost respectively; r is the interest rate; β_1 is an auxiliary variable; σ_p is the volatility for P across the time. We solve for $\hat{P}$ adjusting β_1. The optimal renewal time ($\hat{T}$) is

$$\hat{T}=\frac{1}{\theta_p}\ln\left(\frac{\hat{P}}{P_1}\right) \qquad (5)$$

($\hat{T}$) also represent the economic life of the machine. Under uncertainty, the economic life of a machine is a random variable; therefore, the economic life fluctuates compared to the deterministic model.

2.3 Two factor renewing model

In the two-factor model, we use (Adkins & Paxson, 2011). There is a quasi-analytical approach consisting of a solution of a set of simultaneous equations for problems without having to reduce the dimensions. Specifically, in renewals options, the revenue and the costs are stochastic; both are modeled as Geometric Brownian Motions.

The multi-factor renewal problem is to find the threshold for $\hat{P}$ given $\hat{C}$ (the revenue level P at which the renewal decision is made if the cost level equals $\hat{C}$). The thresholds provide a solution to a set of simultaneous equations. The deterministic model is a special case of the two-factor stochastic model that sets the volatilities values at zero.

The value of the revenue and renewal option is mostly an increasing function of σ_p and σ_c where it is possible to derive analytically. The characteristic root equation, the reduced form value matching relationship and the reduced form smooth pasting condition constitute the two-factor renewal model from which the discriminatory boundary is generated. To determine the boundary, set equations (6), (7), and (8) equal to zero by changing β, η and $\hat{P}$. There is a corresponding $\hat{C}$ that comes from the deterministic model.

The risk-neutral valuation relationship is satisfied by the following characteristic root equation:

$$Q(\beta,\eta) = \frac{1}{2}\sigma_p^2\beta(\beta-1) + \frac{1}{2}\sigma_c^2\eta(\eta-1) + \rho\sigma_p\sigma_c\beta\eta + \theta_p\beta + \theta_c\eta - r = 0 \tag{6}$$

This is the two-factor equivalent of the β quadratic equation for the one-factor model. The smooth pasting:

$$\frac{\hat{P}}{-\beta\left(r-\theta_p\right)} - \frac{\hat{C}}{\eta_4(r-\theta_c)} = 0 \tag{7}$$

And finally, the value matching:

$$H\left(\beta,\eta\middle|\hat{C}\right) = \frac{\hat{C}}{\eta_4(r-\theta_c)}\left(1-\beta-\eta-\frac{P_1^{\beta}C_1^{\eta}}{\hat{C}^{\beta+\eta}}\left(\frac{-\beta\left(r-\theta_p\right)}{\eta(r-\theta_c)}\right)^{-\beta}\right) - \frac{P_1}{r-\theta_p} + \frac{C_1}{r-\theta_c} + K = 0 \tag{8}$$

The variables are the same as in previous models. The deterministic model is a special case for the two-factor model when $\sigma_p = \sigma_c = 0$.

2.4 Technological improvement

(Adkins & Paxson, 2014) extended the two-factor model by adding the technological improvement factor, we specifically present the anticipated technological progress model. Technological advancement is inevitable in businesses and industries. The model assumes the new initial cost of the improved equipment is a growth function with a continuous constant rate. The initial revenue is expected to decrease over time with the new equipment. The Ordinary Differential Equation for this model is:

$$\frac{1}{2}{\sigma_p}^2{P_1}^2\frac{\partial^2 F}{\partial {P_1}^2} + \frac{1}{2}\sigma_c^2{C_1}^2\frac{\partial^2 F}{\partial {C_1}^2} + \frac{1}{2}\sigma_N^2 C_N^2\frac{\partial^2 F}{\partial {C_N}^2} + \rho_{p,c}\sigma_p\sigma_c P_1 C_1\frac{\partial^2 F}{\partial P_1\partial C_1} + \rho_{p,N}\sigma_P\sigma_N P_1 C_N\frac{\partial^2 F}{\partial P_1\partial C_N} + \rho_{c,N}\sigma_c\sigma_N C_1 C_N\frac{\partial^2 F}{\partial C_1\partial C_N} + \theta_p P_1\frac{\partial F}{\partial P_1} + \theta_c C_1\frac{\partial F}{\partial C_1} + \theta_N C_N\frac{\partial F}{\partial C_N} + (P_1 - C) - rF = 0 \tag{9}$$

, where the variables are the same as in previous models, C_N is the technological improvement; θ_N the drift of the improvement; The technological improvement is considered non-stochastic, therefore $\sigma_N = 0$ and the $\rho_{p,N}$, $\rho_{c,N}$ and nil. The function F represent the value matching condition given as:

$$F(P_1, C_1, C_N) = A{P_1}^{\beta}{C_1}^{\eta}C_N^{\gamma} + \frac{P_1}{r-\theta_p} - \frac{C_1}{r-\theta_c} \quad (10)$$

This model has one value matching condition and three smooth pasting conditions, one for each of the variables: P_1, C_1, C_N, and a characteristic root equation:

$$\frac{1}{2}\sigma_p^2\beta(\beta-1) + \frac{1}{2}\sigma_c^2\eta(\eta-1) + \rho_{p,c}\sigma_p\sigma_c\beta\eta$$
$$+\theta_p\beta + \theta_c\eta + \theta_N\gamma - r = 0 \quad (11)$$

This function F is dependent on three factors, the initial operating cost level, the prevailing level of both the revenues and the operating costs of the equipment. $A{P_1}^{\beta}{C_1}^{\eta}C_N^{\gamma}$, with A> 0, represents the option value while $\frac{P_1}{r-\theta_p} - \frac{C_1}{r-\theta_C}$ represents the value of the equipment to the company when there are no options. The variables are the same as in previous models, considering that γ and A as additional auxiliary variables.

The θ_N parameter is firstly estimated. In the second round, it is derived from (12) using different iterations until the value converge. The optimal renewal time ($\widehat{T}$) given by:

$$\widehat{T} = \frac{1}{\theta_N}\ln\left(\frac{C_N}{C_1}\right) \quad (12)$$

3. Study case

The electronic manufacturing industry is highly competitive and demands superior quality products at competitive prices. The Electronics Manufacturing Services (EMS) companies require modern technology and constant control of the revenue and the cost. The EMS offers various services to customers and produces electronics boards with different levels of complexity. The critical process in an EMS is the Surface Mounting Process (SMT), where the electronics components are placed and soldered into the electronics board. For the study case, we selected a server board whose manufacturing process only requires the SMT machine. The server boards have high and constant demand. The production lines work 24hrs 365 days per year, producing the boards. The conversion from machine productivity to revenue is direct; we multiply the assembled units times the unit price.

The SMT process is the bottleneck for the server board. As with any equipment, the SMT lines require higher maintenance through time, reducing their efficiency, i.e., older SMT lines produce fewer units at a higher cost. The revenue decreases and the cost increase; therefore, at some point in time is optimal to renew the machine.

3.1 Data collection

The server board selected is produced constantly during the year; the customer buys all the boards that the EMS can deliver. We collected data from 2016 to 2020 in a quarterly basis, refer to appendix 1. The cloud services companies are the main customers for the server board. Cloud services are a stable and long-term business nowadays. The EMS signed long-term contracts with the customer; the price is commonly fixed for a given period. In the study case, the price per unit for the board is \$17.66, and the initial cost per unit is \$10.08. At a 100% efficiency rate, the SMT lines produce 513,184 units. We compute the initial revenue (P_1= \$9,062,829.44) and the initial cost (C_1= \$5,172,894.72).

The efficiency of the machine defines the revenue. The machine's efficiency decreases over time, producing fewer units. A contract fixes the prices in this study case, but the unit cost increases. The cost of brand-new SMT lines is K = \$3,000,000.00. Getting a new line reverts the efficiency to 100%, restoring the revenue value to its original value. We have the conditions set for a technology renewal problem in the EMS industry.

The revenue (θ_p), cost (θ_c) drifts, and correlation (ρ) were calculated using the five-year quarterly data with the average of the lognormal returns. The volatilities (σ_p, σ_c) follow the same procedure using the standard deviation (θ_p= -0.206; θ_c= -0.123; σ_p= 0.840; σ_c= 0.899; ρ= 0.938). The cost of capital for the company is estimated at 5%; we used this value for the interest rate (r) parameter. We assumed the drifts, volatilities, and interest rate are constant thorough time with no transaction costs or taxes.

The technological improvement model requires an estimation for the renewal drift (θ_N). A more advanced SMT machine reduces the cost \$0.85 per unit. The C_N = \$1,026,368.00. Using (12) and the C_N value we find θ_N= -1.25. The technological improvement is considered non-stochastic, therefore σ_N= 0. The summary of the parameters is presented in Table 1.

Table 1.
Model parameters

Parameter	Value	Parameter	Value
P_1	\$9,062,829.44	σ_p	0.0840
C_1	\$5,172,894.72	σ_c	0.0899
K	\$3,000,000.00	σ_N	0
C_N	\$1,026,368.00	$\rho_{p,c}$	0.938
θ_p	-0.206	$\rho_{p,N}$	-
θ_c	-0.123	$\rho_{c,N}$	-
θ_N	-1.25		

Source: Ikor group (Global EMS)

4. Discussion and results

We implemented the models described in section 2 using the parameters from section 3. The optimal values for revenue and time are denoted as: $\hat{P}$, and $\hat{T}$ respectively. The results and the values of the auxiliary variables to solve the equations are presented in Table 2.

Table 2.
Model results

Variable	Deterministic	One-factor	Two-factor	Technological Improvement
$\hat{\boldsymbol{P}}$	\$ 5,028,939.31	\$ 5,870,557.59	\$ 5,029,515.47	\$6,571,806.06
$\hat{\boldsymbol{T}}$ **(years)**	2.850	2.102	2.849	1.256
β	-0.8373	-0.2370	-0.6753	-0.915
η	0.9999	0.0000	0.7254	0.707
γ	-	-	-	0.042

Source: Models implementation in this paper.

In the deterministic model, the equipment renewal is suggested when the optimal revenue ($\hat{\boldsymbol{P}}$) and the optimal renewal time ($\hat{\boldsymbol{T}}$) is equal to the reversion revenue (P_1) and cost (C_1) values less the investment cost (K). The revenue generated by the SMT machine start at \$9,062,829.44, the results suggest that the company renews the equipment when the revenue generated reaches a level of \$ 5,028,939.31. There is no direct interpretation for the auxiliary variables since their value varies depending on the initial settings. The optimal time to renew ($\hat{\boldsymbol{T}}$) is 2.85 years.

The one-factor model considered the revenue as stochastic; the possibility of fluctuation for the revenue has, as a result, an earlier renewal 2.102 years. In the two-factor model, the revenue and cost are stochastic. The model suggests extending the renewal time is 2.849 years. The volatility for the revenue and the cost is low and similar; there is also a high correlation between them; this explains that the results is similar to the deterministic model. The volatility has a direct impact on the renewal time. Figure 1 shows how changes in the revenue's volatility impact each model's renewal time. The technological improvement model is the most sensitive.

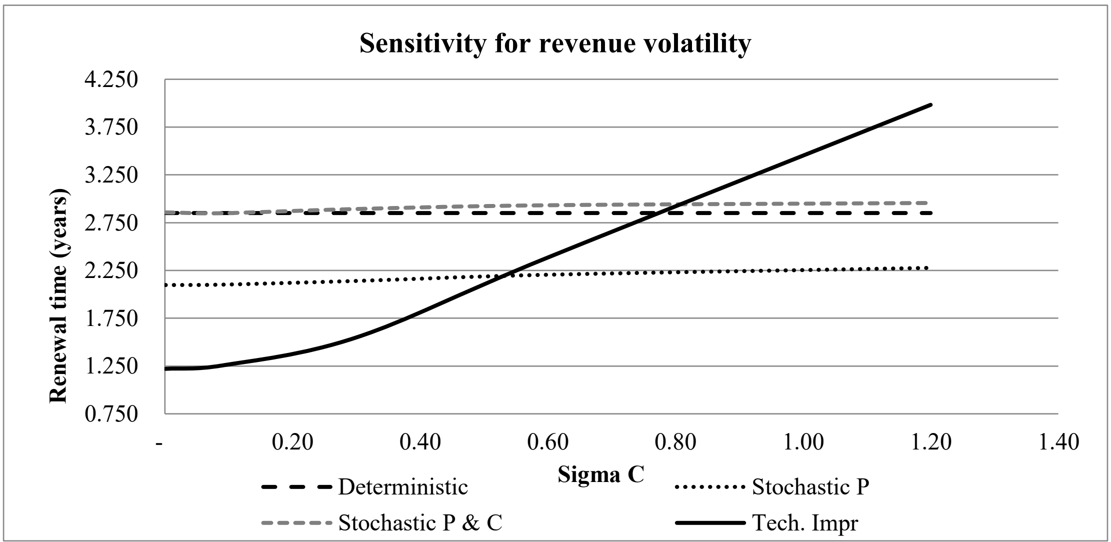

Figure 1: Sensitivity analysis for the volatility of the revenue. The four models are sensitive to changes in the volatility of the revenue; the technological improvement models have the highest impact
Source: Models implementation in this paper.

The lowest renewal time is 1.256 years using the technological improvement model. The results are intuitive; the availability of new equipment in the market that increases the revenue or reduces the cost accelerates the renewal process. Figure 2 presents how the increase in the renewal investment time affects the optimal time to replace equipment. When the cost to buy a new machine is lower, the optimal decision is to renew sooner.

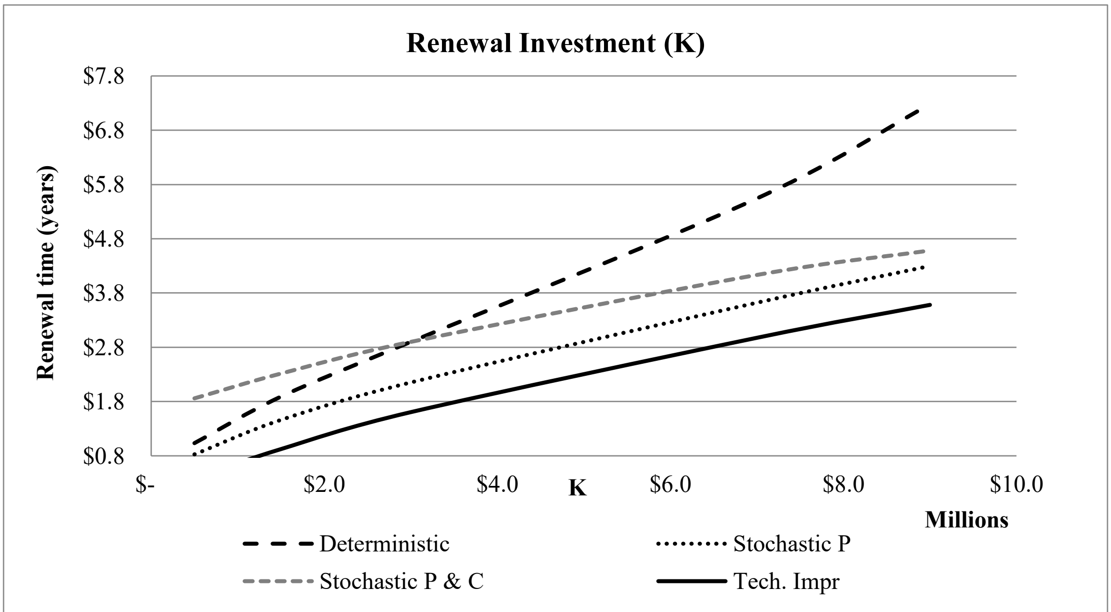

Figure 2: Sensitivity analysis for the renewal investment. The renewal time is sensitive to the renewal investment. The higher the investment, the higher the renewal time
Source: Models implementation in this paper.

5. Conclusion

This paper presents a real options approach to get the optimal renewal time for technology replacement through a study case for an EMS company. We selected four models for implementation: the deterministic model (Linnard & Gane, 1968); the one-factor model (Dobbs, 2004); the two-factor model (Adkins & Paxson, 2011); and the two-factor model with technological progress (Adkins & Paxson, 2014).

The Electronics Manufacturing Services (EMS) companies require modern technology and constant control of the revenue and the cost. There is a continuous decision process to determine the optimal time to replace the old equipment with a new one. The deterministic model provides a baseline to estimate the optimal time to renew equipment; however, in the business environment, the assumption of considering the revenue and the cost without fluctuations doesn't reflect the dynamics of the decision.

The one-factor and two-factor models consider the natural fluctuation of the revenue and the cost in the renewal decision. The study case shows that under certain circumstances, such as low or similar volatility, the results of the models are similar to the deterministic model. The sensitivity analysis demonstrates the impact of volatility in the renewal time. The technological improvement model is the most sensitive to the changes in the volatility of revenue. The renewal investment amount is one of the most relevant factors in the decision. A lower investment leads to a sooner replacement.

It is unlikely that companies acquire the same equipment when they renew; newer machines are faster and more efficient. The technological improvement model considers this factor in the decision. The model is the closest to reality, especially in a sector with continuous technological progression. The results of the study case suggest that the optimal time to replace the SMT machines is between one and three years. The number makes sense in practice, but it is important to adjust the models to the data of each product.

This paper supports the management decision process for the optimal time to renew equipment using real options. Replacing the machines in the optimal time represents a competitive advantage to improve revenue or reduce cost. In future work, we want to explore renewals models that consider tax benefits and depreciation, multidimensional stochastic factors keeping a closed-form solution, and the possibility of a catastrophic failure of the equipment.

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Appendix 1

Year	Qtr.	Efficiency	Unit produced	Price per unit	Cost per unit	Revenue (P)	Cost (C)	Log-returns	
2016	Q1	100.0%	513,184	$17.66	$10.08	$9,062,829.44	$5,172,894.72	p	c
2016	Q2	99.5%	507,836	$17.66	$10.15	$8,968,383.76	$5,154,179.91	-1.05%	-0.36%
2016	Q3	99.0%	502,489	$17.66	$10.20	$8,873,955.74	$5,123,126.60	-1.06%	-0.60%
2016	Q4	98.0%	491,793	$17.66	$10.26	$8,685,064.38	$5,048,156.79	-2.15%	-1.47%
2017	Q1	96.5%	475,750	$17.66	$10.36	$8,401,745.00	$4,927,437.90	-3.32%	-2.42%
2017	Q2	96.0%	470,403	$17.66	$10.54	$8,307,316.98	$4,958,988.43	-1.13%	0.64%
2017	Q3	95.5%	465,055	$17.66	$10.77	$8,212,871.30	$5,010,037.52	-1.14%	1.02%
2017	Q4	95.0%	459,708	$17.66	$11.12	$8,118,443.28	$5,111,723.11	-1.16%	2.01%
2018	Q1	94.0%	449,012	$17.66	$11.86	$7,929,551.92	$5,324,698.60	-2.35%	4.08%
2018	Q2	92.0%	427,621	$17.66	$11.97	$7,551,786.86	$5,120,419.38	-4.88%	-3.91%
2018	Q3	91.0%	416,926	$17.66	$12.16	$7,362,913.16	$5,069,403.23	-2.53%	-1.00%
2018	Q4	90.0%	406,230	$17.66	$12.56	$7,174,021.80	$5,101,537.65	-2.60%	0.63%
2019	Q1	88.0%	384,840	$17.66	$13.01	$6,796,274.40	$5,008,192.31	-5.41%	-1.85%
2019	Q2	85.0%	352,754	$17.66	$13.64	$6,229,635.64	$4,810,647.40	-8.71%	-4.02%
2019	Q3	83.0%	331,363	$17.66	$13.75	$5,851,870.58	$4,557,202.20	-6.26%	-5.41%
2019	Q4	80.0%	299,277	$17.66	$14.06	$5,285,231.82	$4,206,816.89	-10.18%	-8.00%
2020	Q1	78.0%	277,886	$17.66	$14.24	$4,907,466.76	$3,956,540.87	-7.42%	-6.13%
2020	Q2	75.0%	245,800	$17.66	$14.49	$4,340,828.00	$3,562,158.18	-2.27%	-10.50%
2020	Q3	72.0%	213,714	$17.66	$14.63	$3,774,189.24	$3,126,785.42	-13.99%	-13.04%
2020	Q4	70.0%	192,324	$17.66	$14.99	$3,396,441.84	$2,883,743.14	-10.55%	-8.09%